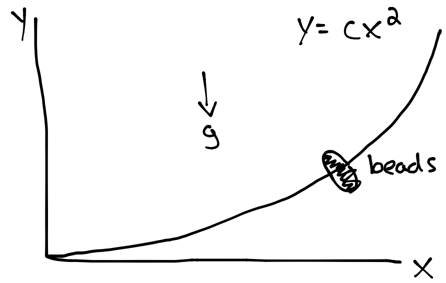


slippery rigid wire

given $x, \dot{x}$

find $\ddot{x}$

<u>Newton Approach</u>

N is perpendicular to the wire

$N \perp \vec{v}$

LMB: $\sum \vec{F} = m\vec{a}$

$\vec{N} - mg\hat{j} = m\vec{a}$ (1)

1 DoF system

<u>1 generalized coordinate</u>: x

$y = cx^2$

Kinematics: $\vec{r} = x\hat{i} + y\hat{j}$

$= x\hat{i} + cx^2\hat{j}$

$\vec{v} = \dot{\vec{r}} = \dot{x}\hat{i} + 2c\dot{x}x\,\hat{j}$

$\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}} = \ddot{x}\hat{i} + (2c\dot{x}^2 + 2cx\ddot{x})\hat{j}$

Take equation (1) and dot it with some vector so that N disappears → Velocity! (they are perpendicular)

$[\vec{N} - mg\hat{j}]\cdot\vec{v} = \cancel{\vec{v}\cdot\vec{N}} - mg\hat{j}\cdot\vec{v} = m\vec{a}\cdot\vec{v}$

$\rightarrow -\cancel{m}g2c\cancel{\dot{x}}x = \cancel{m}\left[\cancel{\dot{x}}\ddot{x} + 4c^2\dot{x}^{\cancel{3}\,2}x + 4c^2x^2\cancel{\dot{x}}\ddot{x}\right]$ solve for $\ddot{x}$

$\rightarrow \quad \ddot{x} = f(x, \dot{x}, c) \rightarrow$ Equation of Motion

method is just like pendulum equation : $\vec{F} = m\vec{a} \cdot$ [vector $\perp$ to constant force]

<u>Lagrangian</u>

$$\mathcal{L} = E_K - E_P = \text{"}T - V\text{"}$$

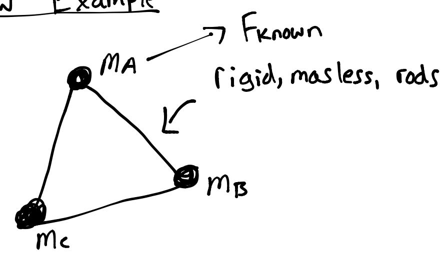
$$= \tfrac{1}{2}mv^2 - mgh = \tfrac{1}{2}m(\dot{x}^2 + 4c^2\dot{x}^2x^2) - mgcx^2$$

$$\mathcal{L}E: \quad \frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} = 0$$

$$\rightarrow m[4c^2\dot{x}^2x - 2gcx] - \frac{d}{dt}[\dot{x} + 4c^2\dot{x}x^2]m = 0$$

$\ddot{x} = f(x, \dot{x}, c)$ ✓   note: for $\frac{\partial \mathcal{L}}{\partial x}$, $\dot{x}$ is a constant

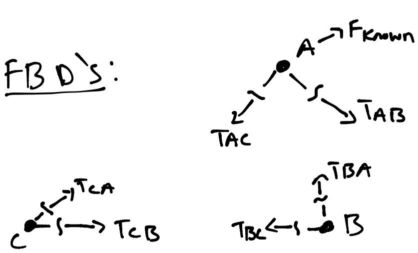
<u>New Example</u>



How does this move?

Set up the DAE's : differential algebraic equations

<u>FBD's</u>:



Kinematic Constraints:

$|\vec{r}_{AB}| = l_{AB} =$ constant

$|\vec{r}_{BC}| =$ "

$|\vec{r}_{CA}| =$ "

LMB:

$\sum_{\text{on}A} \vec{F} = m\vec{a}_A$

. . . .

. . . .

6 scalar equations

for $\ddot{x}_A$, $\ddot{y}_A$

$\ddot{x}_B$, $\ddot{y}_B$

$\ddot{x}_C$, $\ddot{y}_C$

Take Kinematic Constraints

$\vec{r}_{AB} \cdot \vec{r}_{AB} =$ constant

$\frac{d^2}{dt^2}\left[(x_B - x_A)^2 + (y_B - y_A)^2\right] = 0$ → same for other two bars

↳ 2^{nd} derivative because the goal is to get $\ddot{x}\,\ddot{y}$

3 more constants

Total: 9 equations for $\ddot{x}_A, \ddot{y}_A, \ddot{x}_B, \ddot{y}_B$, etc.

and T_{AB}, T_{BC}, T_{AC}

in terms of $\vec{F}_{ext}$, x_A, y_A, x_B . . . .

, $\dot{x}_A$, $\dot{y}_A$, $\dot{x}_B$ . . . . .

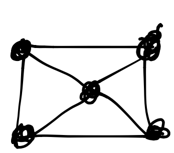
Example: constraint : $x = 7$, → $\ddot{x} = 0$

I.C.'s $x_0 = 7$

$\dot{x}_0 = 0$

→ $\ddot{x} = 0$ for $x = 7$ for all t

Example:



LMB gives 10 eq's

constraint eq's give 8

total = 18 eqs → 10 unknown accelerations

8 unknown tensions